Theoretical Research of Weak Neutron Source Induced Bursts in Fast Neutron Reactor:

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Abstract: We find that theoretical calculation using Hansen's model is significantly different from experimental results of weak source induced busts by Wimett et al on Godiva-II. Based on solution of the probability of n neutrons at time t, the expected length of finite fission chain is evaluated and the multiplication of delayed neutron precursors is studied, which indicate that the number of delayed neutrons may vary several times during the waiting time. With consideration of the multiplication of delayed neutron precursors, an improved theoretical model of probability distributions of burst wait-time is presented, which consists well with experimental results of Godiva-II

Key words: Pulsed Reactor; the probability distribution of n neutrons at t; Non-Persistent Fission Chains; Delayed Neutron Precursors; probability distributions of burst wait-time。

I. INTRODUCTION

In slightly supercritical system in presence of weak source, the stochastic behavior of neutrons during multiplication has attracted much attention for many years.

The stochastic behavior of neutrons had been shown by Wimett et al. on Godiva-II in 1960[1], and was also confirmed on CFBR-II by the Institute of Nuclear Physics and Chemistry, China Academy of Engineering Physics.

Hansen studied this phenomena and developed a theoretical model which has been used by many other researchers since then. However, we note that when the average waiting time in presence of weak source and the probability distributions in time of burst occurrence are calculated using Hansen's method, the consistency between calculations and experimental results is not good.

This stochastic behavior was still not well understood till 2007, as pointed by Greenman, LLNL believed it is necessary to develop new code in order to simulate this behavior of neutrons in experiments.

We studied Hansen's model and found that the contributions of finite fission chains and delayed neutrons are ignored in his model, although these contributions have been discussed in his paper. Furthermore, the source strength during the waiting time and the probability of a source neutron sponsoring a persistent fission chain are also considered as constants in Hansen's model. In this paper, we improve Hansen's model by solving the expectation of finite fission chain and the multiplication of delayed neutron precursors due to finite fission chain. The calculated results are in good agreement with the experimental data.

II. PROBABILITY EQUILIBRIUM EQUATION $P_n(t_0,t)$ AND ITS SOLUTION

We discuss the probability of a source neutron sponsoring a persistent fission chain. The probability equilibrium equation and its solution are given.

We consider a simple point reactor in which all neutrons behave identically, each neutron has the probability p of producing fission, each fission has the probability p(ν) of emitting ν neutrons. Let W be the probability of a source neutron sponsoring a persistent fission chain. Denote $P_n(t_0,t)$, the probability of a source neutron at time t0 sponsoring n neutrons at time t, which satisfies the equation

$$\begin{aligned}\tau\frac{dP_n(t_0,t)}{dt} &= (1-p)(n+1)P_{n+1}(t_0,t) - nP_n(t_0,t) \\ &+ p\sum_{\nu=0}^{\infty}P(\nu)(n-\nu+1)P_{n-\nu+1}(t_0,t)\end{aligned} \quad . \tag{1}$$

Here τ is the mean lifetime of a neutron. The initial condition of $P_n(t_0,t)$ is

$$P_n(t_0,t) = \delta_{nl} \qquad \delta_{nl} = \begin{cases} 1 & n = l \\ 0 & n \neq l \end{cases} \tag{2}$$

Where δ_{nl} is the Kronecker delta. Obviously $P_n(t_0,t)$ satisfies

$$\sum_{n=0}^{\infty} P_n(t_0,t) = 1 \, . \tag{3}$$

By introduction of parameter *z*, the generation function of $P_n(t_0,t)$ can be defined as

$$G(z;t_0,t) = \sum_{n=0}^{\infty} P_n(t_0,t) z^n \, . \tag{4}$$

So we can have

$$P_0(t_0,t) = 1 - \frac{\varepsilon}{1+\eta} \tag{5}$$

$$P_n(t_0,t) = \frac{\varepsilon}{(1+\eta)\eta}\left(1+\frac{1}{\eta}\right)^{-n} \qquad (n \neq 0) . \tag{6}$$

With $\alpha = \frac{1}{\tau}(k-1)$, $k = \bar{\nu}p$, $\bar{\nu}$ is the average number of neutrons emitted by one

$$\varepsilon = e^{\frac{1}{\tau}(k-1)(t-t_0)} = e^{\alpha(t-t_0)} \tag{7}$$

$$\eta = \frac{\Gamma_2 \bar{\nu}}{2\rho}\left(e^{\frac{1}{\tau}(k-1)(t-t_0)} - 1\right) = \frac{\Gamma_2 \bar{\nu}}{2\rho}\left(e^{\alpha(t-t_0)} - 1\right) \tag{8}$$

where $\rho = \frac{k-1}{k}$ is the prompt reactivity. Thus, as we can see the probability W of a source neutron sponsoring a persistent fission chain is related to the prompt reactivity ρ and the mean lifetime τ of a neutron. And (8) can be rewrited as

$$\eta = \frac{\Gamma_2 \overline{\nu}}{2\rho} \left(e^{\alpha(t-t_0)} - 1\right) = \frac{1}{W}\left(\varepsilon - 1\right). \quad (9)$$

When η is large enough, n in) can be considered as continuous, thus we have

$$P_n(t_0, t) = \frac{\varepsilon}{(1+\eta)\eta} e^{-\frac{n}{\eta}}. \quad (10)$$

III. THE PROBABILITY OF EXTINCTION AND SPONSORING A PERSISTENT FISSION CHAIN AT TIME T

Deduction of the probability of extinction and sponsoring a persistent fission chain at time t is presented.

We assume the first neutron is introduced into the system at $t_0 = 0$. And the probability of n neutrons sponsoring persistent fission chain is $1 - e^{-nW}$, where e^{-nW} is the probability of n neutrons not sponsoring persistent fission chain. Thus, $n\nu\Sigma_f P_n(t)(1 - e^{-nW})dt$ is the contribution of the probability of n neutrons in the system to sponsoring a persistent fission chain in dt, whereas $n\nu\Sigma_f P_n(t) e^{-nW} dt$ is the contribution of the probability of n neutrons in the system to sponsoring a non-persistent fission chain in dt.

Define $P_0(t)$ as the extinction probability at time t , and assume there is already one neutron in the system at t = 0, thus $P_0(0) = 0$ is the initial condition. And we also have

$$P_n(t) = \frac{\varepsilon}{(1+\eta)\eta} e^{-\frac{n}{\eta}} \quad (11)$$

where $\varepsilon = e^{\alpha t}$, $\eta = \frac{1}{W}\left(e^{\alpha t} - 1\right)$, $P_0(t) + \int_0^\infty P_n(t) dn = 1$.

The neutrons sponsoring persistent fission chain will not die out, that is

$$\int_0^\infty P_n(t)\left(1 - e^{-nW}\right) dn = P_1(0)\left(1 - e^{-W}\right) = W. \quad (12)$$

The extinction probability at time t equals the probability of sponsoring non-persistent fission chain at the initial time minus these at time t, that is,

$$P_0(t) = (1-W) - \int_0^\infty P_n(t) e^{-nW} dn \tag{13}$$

$$P_0(t) = 1 - \frac{\varepsilon}{1+\eta} = 1 - \frac{e^{\alpha t}}{1+\frac{1}{W}\left(e^{\alpha t}-1\right)} \tag{14}$$

$$\int_0^\infty P_n(t) e^{-nW} dn = \frac{\varepsilon}{(1+\eta)\eta} \int_0^\infty e^{-\frac{n}{\eta}} e^{-nW} dn$$

$$= \frac{\varepsilon}{(1+\eta)\eta} \frac{1}{-\left(\frac{1}{\eta}+W\right)} \int_0^\infty e^{-\left(\frac{1}{\eta}+W\right)n} d\left[-\left(\frac{1}{\eta}+W\right)\right] n$$

$$= -\frac{e^{\alpha t}}{1+\frac{1}{W}(e^{\alpha t}-1)} \frac{1}{1+e^{\alpha t}-1} = \frac{1}{1+\frac{1}{W}(e^{\alpha t}-1)} \tag{15}$$

$$\int_0^\infty P_n(t) dn = \int_0^\infty \frac{\varepsilon}{(1+\eta)\eta} e^{-\frac{n}{\eta}} dn = -\frac{\varepsilon}{(1+\eta)} \int_0^\infty e^{-\frac{1}{\eta}n} d\left(-\frac{1}{\eta}\right) n$$

$$= \frac{\varepsilon}{(1+\eta)} = \frac{e^{\alpha t}}{1+\frac{1}{W}\left(e^{\alpha t}-1\right)} \tag{16}$$

Substitution of (15) and (16) into (12) yields

$$\int_0^\infty P_n(t)\left(1-e^{-nW}\right) dn = \int_0^\infty P_n(t) dn - \int_0^\infty P_n(t) e^{-nW} dn$$

$$= \frac{e^{\alpha t}}{1+\frac{1}{W}\left(e^{\alpha t}-1\right)} - \frac{1}{1+\frac{1}{W}(e^{\alpha t}-1)} = \frac{e^{\alpha t}-1}{1+\frac{1}{W}(e^{\alpha t}-1)} = \frac{W}{\frac{W}{e^{\alpha t}-1}+1} \approx W \tag{17}$$

which is satisfied under the condition $W/\left(e^{\alpha t}-1\right) << 1$, or $\left(e^{\alpha t}-1\right) >> W$.

From the experimental data of Godiva-II, $W \approx 10^{-3}$, and the above condition requires $\alpha t >> 10^{-3}$ or $t >> \frac{10^{-3}}{\alpha}$.Apparently, $t < \frac{10^{-3}}{\alpha}$ is a very short amount of time, which is also the requirement in the deduction of the continuous form of $P_n(t_0, t)$, where

$\frac{1}{\eta}$<<1, or $W/\left(e^{\alpha t}-1\right)$<<1 is required.

From (15) and (16), we obtain the left side of eq. (13)

$$P_0(t)=1-\frac{e^{\alpha t}}{1+\frac{1}{W}\left(e^{\alpha t}-1\right)}, \tag{18}$$

and the right side of eq. (13)

$$1-W-\int_0^\infty P_n(t)e^{-nW}dn=1-W-\frac{1}{1+\frac{1}{W}(e^{\alpha t}-1)}$$

$$=1-\frac{W+e^{\alpha t}}{1+\frac{1}{W}\left(e^{\alpha t}-1\right)} \tag{19}$$

respectively. We note that $e^{\alpha t}>>W$ is satisfied for Godiva-II，thus (18) equals (19) approximately. Therefore, eqn. (13) is verified.

IV. EXPECTATION OF FINITE FISSION CHAIN LENGTH $\langle n_f \rangle$

We evaluate the expectation of the finite fission chain and give the caculated，

Let us consider a point reactor and ignore all delayed neutrons, assume one neutron is introduced into the system at the starting point which sponsors a non-persistent fission chain. Then the average number of fissions $\langle n_f \rangle$ is defined as the expectation of the finite fission chain length . As defined in section III, e^{-nW} is the probability of n neutrons not sponsoring persistent fission chain, and $n\nu\Sigma_f P_n(t)e^{-nW}dt$ is the contribution of the probability of n neutrons in the system sponsoring a non-persistent fission chain in dt, we have

$$\langle n_f \rangle=\int_0^\infty dn\int_0^\infty n\nu P_n(t)e^{-nW}dt \tag{20}$$

we have

$$\langle n_f \rangle=\int_0^\infty dtG=\Sigma_f\nu\int_0^\infty dt\frac{\varepsilon}{(1+\eta)\eta}\frac{1}{\left(\frac{1}{\eta}+W\right)^2}=\Sigma_f\nu\int_0^\infty dt\frac{e^{\alpha t}}{e^{2\alpha t}}=\frac{\Sigma_f\nu}{\alpha}=\frac{1}{\bar{\nu}\rho}. \tag{21}$$

Here $\frac{\eta}{(1+\eta)} \approx 1$ is assumed. Let $\langle n_f \rangle = \Sigma_f \nu \int_0^T dt e^{-\alpha t} = \frac{1}{\nu\rho}\left(1-e^{-\alpha T}\right)$, $\alpha T = 3$, then $\langle n_f \rangle_{\alpha T=3} = \frac{1}{\bar{\nu}\rho} 0.95$。which means the relaxation time of forming a finite fission chain is $T = \frac{3}{\alpha}$. The physical meaning of eq.(52) is manifest, for ρ<<1, $\langle n_f \rangle$ is inversely proportion to $\bar{\nu}\rho$.

Now we can evaluate the expectation of the fission numbers of finite fission chain. Given βeff=0.0069, $\bar{\nu}$=2.59, Γ2=0.795, ρ (\$)=0.05 for Godiva-Ⅱ，the expectation is $\langle n_f \rangle \approx 1119$。which is well consistent with $\langle n_f \rangle \approx 1200$ from Spriggs [3].

V DELAYED NEUTFRON PRECURSORS EQUATION AND ITS SOLUTION

The multiplication of delayed neutron precursors is studied, which indicates that the number of delayed neutrons may vary several times during the waiting time.

Under the point reactor model, the differential equations for the slow development of the delayed neutron precursor due to the finite fission chain are considered as;

$$\frac{dC_i(t)}{dt} = -\lambda_i C_i(t) + [S_0 + \sum_{i=1}^{6} \lambda_i C_i(t)]\bar{\nu}\beta_i \langle n_f(t) \rangle \alpha_i \tag{22}$$

$$\sum_{i=1}^{6} \alpha_i = 1 \tag{23}$$

Among them, $C_i(t)$ is fraction of the delayed neutrons precursors , the i group ; λ_i is the constant disntegration of the i group of delayed neutrons precursors fraction,

$\langle n_f(t) \rangle$ is the mathematical expectation of the fission times of a source neutron and its progeny， S_0 is the Spontaneous fission source and initial delayed neutron source

In $S_0 = S_{sf} + S_d$， S_{sf} is the spontaneous fission source for the device， S_d is the Delayed neutron source。If there is the delayed neutron in the initial system S_d， The spontaneous fission neutron can be regarded as the initial source S_0.

According to the formula (23), it is known that the $\langle n_f(t) \rangle$ slow variation of the neutron with time is related to the expectation of the finite length fission chain. When the formula (1) is made of a single group of delayed neutrons, the neutron source in the system can be expressed as:

$$S(t) = S_0 + \xi C(t) \tag{24}$$

Among them, the mathematical expectation of the concentration of the precursor is the average decay constant of the single group.
$C(t)$ will meet the following differential equation;

$$\frac{dC(t)}{dt} = -\xi C(t) + [S_0 + \xi C(t)]\bar{\nu}\beta_{eff}\langle n_f(t)\rangle \tag{25}$$

Obviously, it is also needed to consider prompt reactivity ρ_P and the delayed reactivity of finite fission chain expectation value contribution, therefore, $\langle n_f(t)\rangle$ meet the following relationships:

$$\langle n_f(t)\rangle = \frac{1}{\bar{\nu}\rho_P + \bar{\nu}\beta_{eff}\left(1 - e^{-\xi t}\right)} \tag{26}$$

Handle (26) into equation (25) obtained

$$\frac{dC(t)}{dt} = -\xi C(t) + [S_0 + \xi C(t)]\ \frac{\beta_{eff}}{\rho_P + \beta_{eff}\left(1 - e^{-\xi t}\right)} \tag{27}$$

If the initial system does not slow the neutrons, so there are initial conditions

$$C(t = 0) = 0 \tag{28}$$

Solving equation (27), and obtained by the initial condition (28)

$$C(t) = \frac{\beta_{eff}}{\xi\rho_P}\left(1 - e^{-\xi t}\right)S_0 \tag{29}$$

Handle (29) into equation (24) obtained

$$S(t) = S_0\left[1 + \frac{\beta_{eff}}{\rho_P}\left(1 - e^{-\xi t}\right)\right] \tag{30}$$

According to the formula (30) and the experimental parameters, the proliferation of delayed neutrons in the waiting time of the pulse is calculated.
for example: When the $t = 0$， $S(0) = S_0$。

The results of delayed neutron multiplication with the proliferation of Godiva-II were included in Table 1 during the waiting period.

Table 1 the proliferation data of delayed neutron in Godiva-II reactor

t(sec)	0	1	2	3	4	5	6	7	8
S(t)(1/sec)	90	690	1090	1357	1653	1732	1732	1784	1820

When the waiting time $t \to \infty$:

$$S(t) = S_0\left[1 + \frac{\beta_{eff}}{\rho_P}\right] \tag{31}$$

The experimental parameter of Godiva-II given by the Wimett
$\beta_{eff} = 0.0069$, $\rho_P = 0.05$, $S_0 = S_{sf} \approx 90/\sec$ the source of the source of strength.
According to the experimental parameters, and formula(68), the maximum can be calculated to increase the system's delayed neutrons to more than 20 times.

VI. An Improved Model

The Hansen's model is improved by introducing the finite fission chain and delayed neutron precursors. An improved theoretical model of probability distributions of burst wait-time is presented, which consists well with experimental results of Godiva-II

The probability distribution model of waiting time probability distribution of pulsed reactor weak source is given under the assumption of step approximation of Hansen.

$$P_{1st}(t_1)dt_1 = \exp(-WSt_1)WSdt_1 \quad (32)$$

After the previous discussion, we think that the delayed neutrons will be in the system during the waiting period. This will cause W to change, that is $\rho = \rho(t)$ 、 $W = W(t)$.

Therefore, the Hansen model (31) can be rewritten as;

$$P_{1st}(t_1)dt_1 = W(t_1)S(t_1)dt_1 \exp\left[-\int_0^{t_1} W(t)S(t)dt\right] \quad (33)$$

Obviously, formula (33) is difficult to obtain analytical solution directly. In order to solve, this order, then formula (33) can be expressed as

$$P_{1st}(t_1)dt_1 = F(t_1)dt_1 \exp(-\int_0^{t_1} F(t)dt \quad (34)$$

$$F(t) = W_0 S_{cv}(t) \quad (35)$$

The implication is that the spontaneous fission neutron and delayed neutron is equivalent to neutron source. The type (35) into equation (34)

$$P_{1st}(t_1)dt_1 = F(t_1)dt_1 \exp(-\int_0^{t_1} F(t)dt$$

$$= W_0 S_{cv}(t_1)dt_1 \exp\left[-\int_0^{t_1} W_0 S_{cv}(t)dt\right] \quad (36)$$

Before previous chapter . We've got slow neutron $S_{cv}(t)$ expressions (30), formula (30) direct substitution type (36), and pay attention to this point, $W_0 \cong \frac{2\rho_p}{\bar{\nu}\Gamma_2}$, we can get improved Hansen burst probability distribution model for:

$$P_{1st}(t_1)dt_1 = dt_1 S_0\left[\frac{2}{\bar{\nu}\Gamma_2}(\rho_P + \beta_{eff}) - \frac{2}{\bar{\nu}\Gamma_2}\beta_{eff}e^{-\xi t_1}\right]$$
$$\times \exp\left[-S_0\left(\frac{2}{\bar{\nu}\Gamma_2}(\rho_P + \beta_{eff})\right)t_1 + S_0\frac{1}{\xi}\frac{2}{\bar{\nu}\Gamma_2}\beta_{eff}\left(1 - e^{-\xi t_1}\right)\right] \quad (37)$$

Using the formula (37) and the experimental parameters, the fitting curve and the experimental results are satisfactory, as shown in Figure

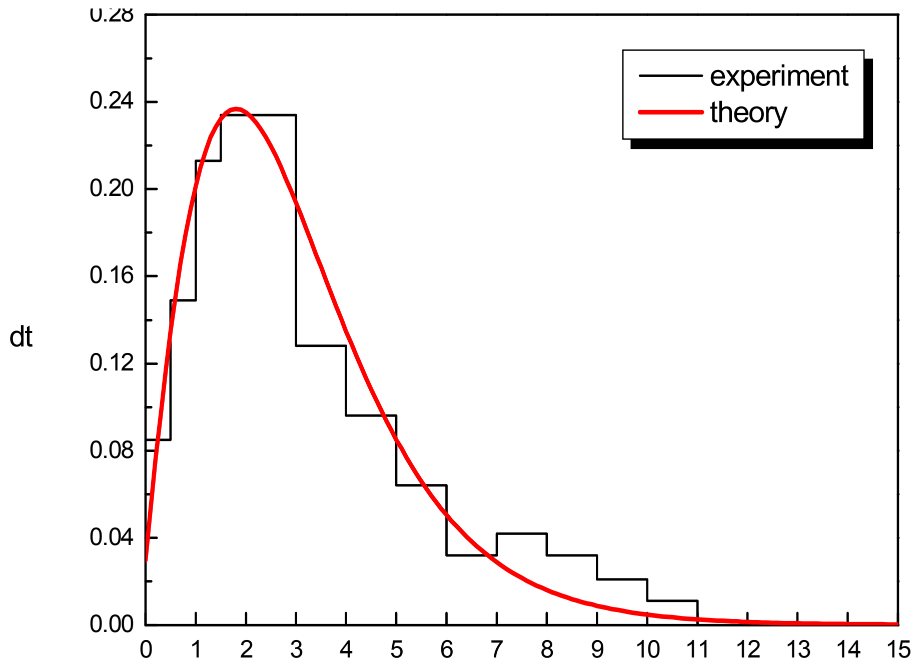

Fig. probability distribution curve of waiting time for Godiva reactor with 94 times

VII CONCLUSION

In the study of fast neutron pulsed reactor of weak source induced busts experiment, we found the limitations of Hansen's model and method of the Wimett, Hansen's model is improved by introducing the finite fission chain and delayed neutron precursors, the model can be a very good description of the experimental results. The model is also applicable to other similar experiments of pulsed reactor.

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